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Solution to 2017-3 Problem 3

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Abstract. 2017-3 Problem 3 is generalized.

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1. Problem 3

We generalize the following problem in [1]:

Problem 1. Let ABC be an isosceles triangle with base BC, and let $DEFG$ be a square such that D, E and FG lie on the sides AB, CA and BC, respectively, and $DE = a$. $PQRG$ is a square such that P, Q and R lie on the sides DG, EB and BG, respectively, and $PQ = b$; $STUV$ is a square such that S, T, and UV lie on the sides AD, EA and DE, respectively, and $ST = b$. Find a in terms of b.

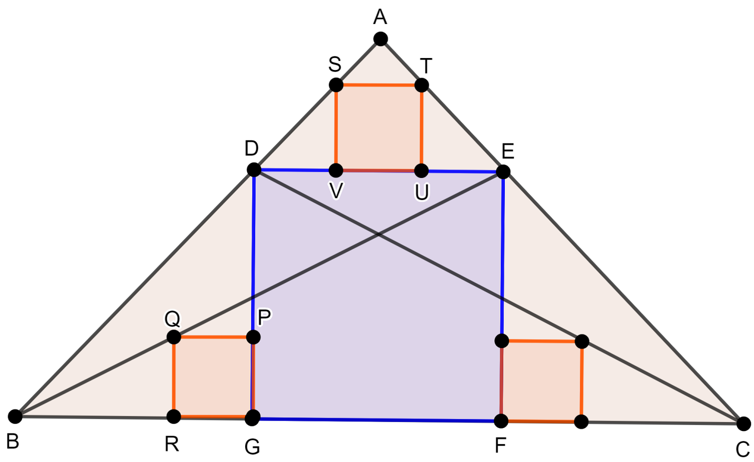

Figure 1. The problem.

2. Generalization

We generalize the problem.

Theorem 1. *Let ABC be an isosceles triangle with base BC, and let $DEFG$ be a square such that D, E and FG lie on the sides AB, CA and BC, respectively, and $DE = a$. If $PQRG$ is the rectangle such that P, Q and R lie on the sides DG, EB and BG, respectively, and $GP = b$ and $PQ = nb$, and $STUV$ is the square such that S, T and UV lie on the sides AD, EA and DE, respectively, and $ST = b$, then $a = (1 + \sqrt{2n+2})b$ holds.*

Proof. Let H be the foot of perpendicular from A to BC, $h = AH$ and $l = BH$. From the similar triangles ABH, DBG, SDV, we have the following equations:

$$\frac{h}{l} = \frac{a}{l - \frac{a}{2}} = \frac{b}{\frac{a}{2} - \frac{b}{2}}. \tag{1}$$

From the similar triangles EBF and QBR, we have

$$\frac{a}{l + \frac{a}{2}} = \frac{b}{l - \frac{a}{2} - nb}. \tag{2}$$

Eliminating h and l from the equations (1) and (2), and solving the resulting equation for a, we get $a = (1 + \sqrt{2n+2})b$. □

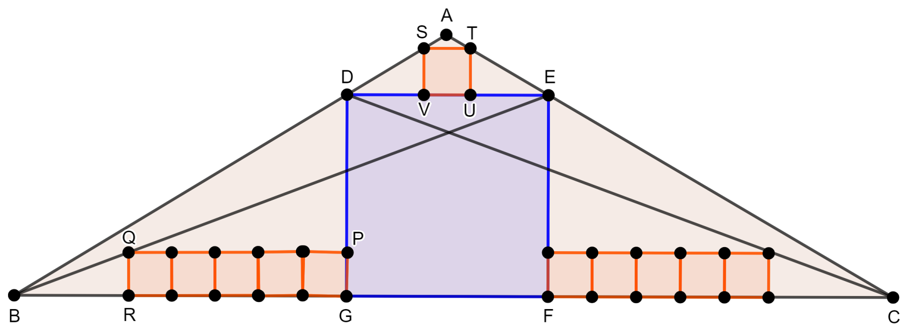

Figure 2. $n = 5$.

Remark. The equations (1) yield the following rations:

$$h = \frac{a^2}{a-b}, \quad l = \frac{a^2}{2b}.$$

References

[1] Problems 2017-3, Sangaku J., Math., (2017) 21-23.