

Problems 2019-2

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We propose four problems. Problem 1 is taken from the sangaku in Yamagata hung in 1881 proposed by Gotoh (後藤友七光介) [1]. Please send a solution with an extra result or a generalization.

We assume that $ABCD$ is a square, E and F are the points on the sides CD and DA , respectively, such that BEF is an equilateral triangle, G is the midpoint of BE , H in the point of intersection of the sides EF and DG and I is the point of intersection of the sides AG and BF for Problems 1, 2, 3, 4.

Problem 1. Prove or disprove that

- (i) AGD is an equilateral triangle (see Figure 1),
- (ii) the inradius of the triangle DFH is double the inradius of the triangle ABI .

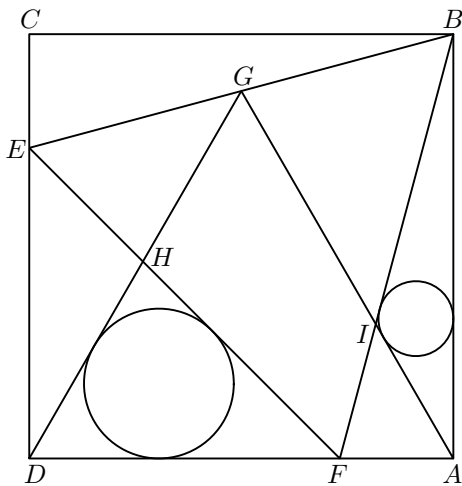


Figure 1.

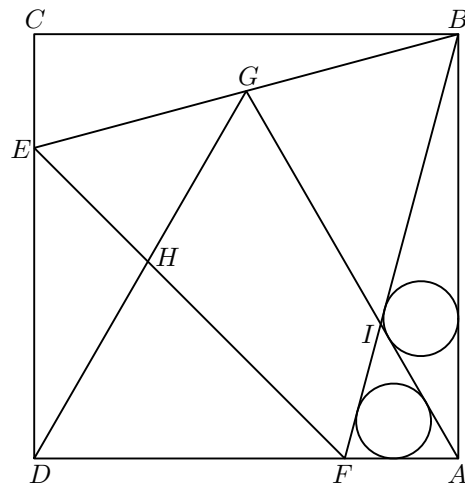


Figure 2.

Problem 2. Prove or disprove that the incircles of the triangles ABI and AIF are congruent (see Figure 2).

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Problem 3. Prove or disprove that the circumcircle of the triangle EHG touches the lines BC , CD and AG at G (see Figure 3).

Remark. If Problem 3 is proved, then the circumcircle of EHG is also one of the excircles of the triangle formed by the three lines.

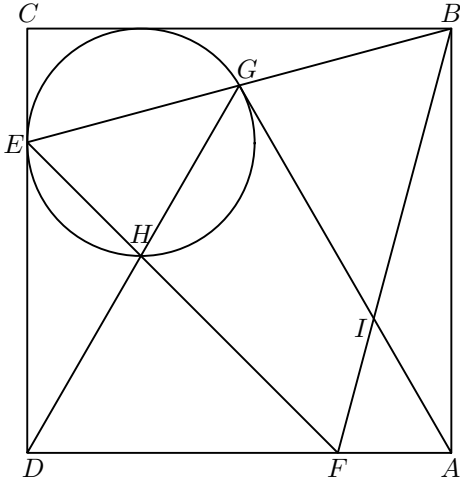


Figure 3.

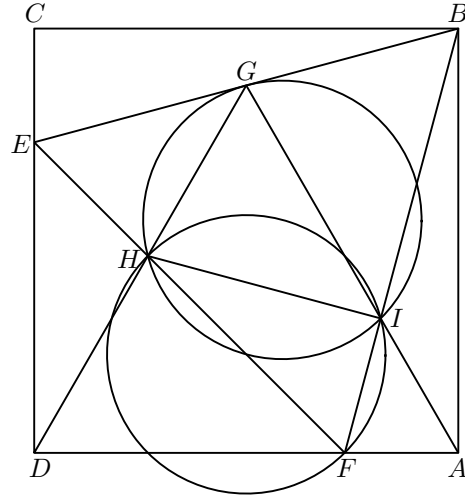


Figure 4.

Problem 4. Prove or disprove that the circumcircle of the triangle FIH is the reflection of the circumcircle of the triangle GHI in the line HI (see Figure 4).

REFERENCES

- [1] A. Hirayama, M. Matsuoka ed., The sangaku in Yamagata, 1966.