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# A simple method to explore the relationships between six areas of a cevasix configuration

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**Abstract.** If $P$ is a point inside $\triangle ABC$, then the cevians through $P$ divide $\triangle ABC$ into six small triangles. We introduce a simple method to explore the relationships between the areas of these triangles.

**Keywords.** triangles, cevians, areas, cevasix configuration

**Mathematics Subject Classification (2010).** 51M04.

## 1. Introduction

Let $P$ be a point inside $\triangle ABC$, then the cevians through $P$ divide $\triangle ABC$ into six small triangles. Let the areas of the six small triangles be $K_1$ to $K_6$ as shown in Figure 1 and let the area of $\triangle ABC$ be K. Rabinowitz explored the relationships between the areas of the six triangles in [1]. He discovered the relationships with the help of computers. In this paper, we give a simple approach to explore the relationships between the six small triangles that does not require the use of computers.

## 2. Expressing $K_4$, $K_5$, $K_6$ and $K$ in terms of $K_1$, $K_2$, $K_3$

Let the areas of $\triangle PBC$, $\triangle PCA$ and $\triangle PAB$ be $x$, $y$ and $z$ respectively. Note that $AF : FB = y : x$, $BD : DC = z : y$, $CE : EA = x : z$. We can express the areas of six small triangles and the area of $\triangle ABC$ in terms of $x$, $y$ and $z$. The expressions are as follows: $K_1 = \frac{xz}{y+z}$, $K_2 = \frac{xy}{y+z}$, $K_3 = \frac{xy}{x+z}$, $K_4 = \frac{yz}{x+z}$, $K_5 = \frac{yz}{x+y}$, $K_6 = \frac{xz}{x+y}$, $K = x + y + z$.

Solving the first three equations gives the following results.

$$x = K_1 + K_2, \tag{1}$$

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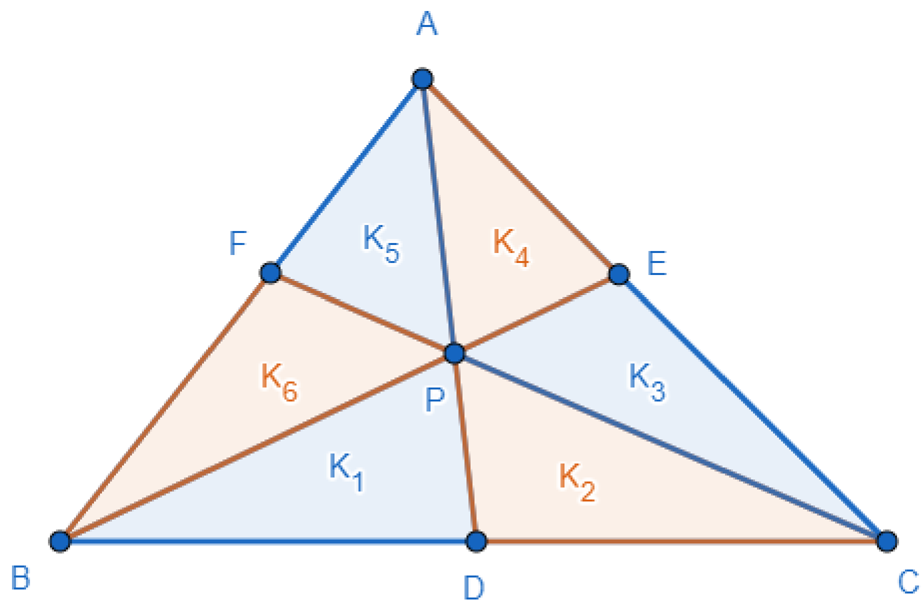


FIGURE 1. The cevasix configuration

$$y = \frac{K_2K_3(K_1+K_2)}{K_1K_2 - K_1K_3 + K_2^2}, \tag{2}$$

$$z = \frac{K_1K_3(K_1+K_2)}{K_1K_2 - K_1K_3 + K_2^2}. \tag{3}$$

By (1), (2), (3) and the expressions of $K_4$, $K_5$, $K_6$ and $K$ in terms of $x$, $y$ and $z$, we have:

$$K_4 = \frac{K_1K_3^2}{K_1K_2 - K_1K_3 + K_2^2}, \tag{4}$$

$$K_5 = \frac{K_1K_2K_3^2(K_1+K_2)}{(K_1K_2 - K_1K_3 + K_2^2)(K_1K_2 - K_1K_3 + K_2^2 + K_2K_3)}, \tag{5}$$

$$K_6 = \frac{K_1K_3(K_1+K_2)}{K_1K_2 - K_1K_3 + K_2^2 + K_2K_3}, \tag{6}$$

$$K = \frac{K_2(K_1+K_2)(K_1+K_2+K_3)}{K_1K_2 - K_1K_3 + K_2^2}. \tag{7}$$

(7) is the solution for the problem on a wooden tablet hung by Sugita Naotake in the Izanagi shrine in the Mie Prefecture of Japan in 1835 [2].

## 3. Expressing $K_3$, $K_5$, $K_6$ and $K$ in terms of $K_1$, $K_2$, $K_4$

Similarly, we can use this technique to discover formulae which express $K_3$, $K_5$, $K_6$ and $K$ in terms of $K_1$, $K_2$, $K_4$.

By solving the equations $K_1 = \frac{xz}{y+z}$, $K_2 = \frac{xy}{y+z}$ and $K_4 = \frac{yz}{x+z}$, we have the following results:

$$x = K_1 + K_2, \tag{8}$$

$$y = \frac{\sqrt{K_1}K_4 + \sqrt{K_4}\omega}{2\sqrt{K_1}}, \tag{9}$$

$$z = \frac{K_1K_4 + \sqrt{K_1K_4}\omega}{2K_2}, \tag{10}$$

where $\omega = \sqrt{4K_1K_2 + K_1K_4 + 4K_2^2}$.

By (8), (9), (10) and the expressions of $K_3$, $K_5$, $K_6$ and $K$ in terms of $x$, $y$ and $z$, we have:

$$K_3 = \frac{K_2(K_1 + K_2)(\sqrt{K_1}K_4 + \sqrt{K_4}\omega)}{\sqrt{K_1}(2K_1K_2 + 2K_2^2 + K_1K_4 + \sqrt{K_1K_4}\omega)}, \tag{11}$$

$$K_5 = \frac{\sqrt{K_1}K_4(\sqrt{K_1K_4} + \omega)^2}{2K_2(2K_1^{\frac{3}{2}} + 2\sqrt{K_1}K_2 + \sqrt{K_1}K_4 + \sqrt{K_4}\omega)}, \tag{12}$$

$$K_6 = \frac{K_1\sqrt{K_4}(K_1 + K_2)(\sqrt{K_1K_4} + \omega)}{K_2(2K_1^{\frac{3}{2}} + 2\sqrt{K_1}K_2 + \sqrt{K_1}K_4 + \sqrt{K_4}\omega)}, \tag{13}$$

$$K = \frac{(K_1 + K_2)(2\sqrt{K_1}K_2 + \sqrt{K_1}K_4 + \sqrt{K_4}\omega)}{2\sqrt{K_1}K_2}. \tag{14}$$

## References


[1] Stanley Rabinowitz, *Relationships Between Six Areas*. Sangaku Journal of Mathematics. 4(2020), 77-83.

[2] Hidetosi Fukagawa and Tony Rothman, *Sacred Mathematics: Japanese Temple Geometry*. Princeton University Press, 2008.