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Solution to 2018-1 Problem 2

Viera Čerňanová
Trnava University, Faculty of Education,
Department of Mathematics and Computer Science,
Priemyselná 4, 918 43 Trnava, Slovakia
e-mail:vieracernanova@hotmail.com

Abstract. A geometric solution to Problem 2 (2018-1) and a new thought are given.

Keywords. sangaku, equilateral triangles, reflection, trapezoid, incircles.

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Problem 2. B is a point on the segment $\overline{AC}$ (see Figure 1), X and Y are points lying on the same side of the line AC such that ABX and BCY are equilateral triangles. Prove or disprove $AY = CX$.

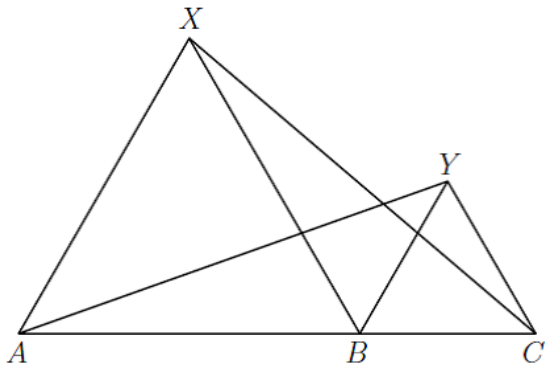

Figure 1.

Solution. Denote Y' the reflection of Y across the line AB (see Figure 2). Since A, B, C are collinear, and ABX and BCY are equilateral, $AXCY'$ is a trapezoid with equal diagonals $AC = XY'$. Thus $AXCY'$ is isosceles, and also

$$AY = AY' = XC.$$

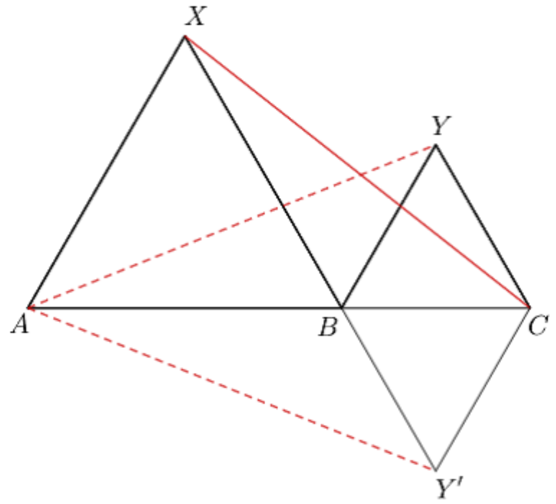

FIGURE 2.

Special case and solution. Configurations in Wasan geometry often contain incircles. Figure 3 shows such configuration, where each circle is tangent to one side and both diagonals of an isosceles trapezoid, and three of circles are congruent. If a, b are the lengths of the bases and c is that of the legs of the trapezoid, find the ratio $a : b : c$.

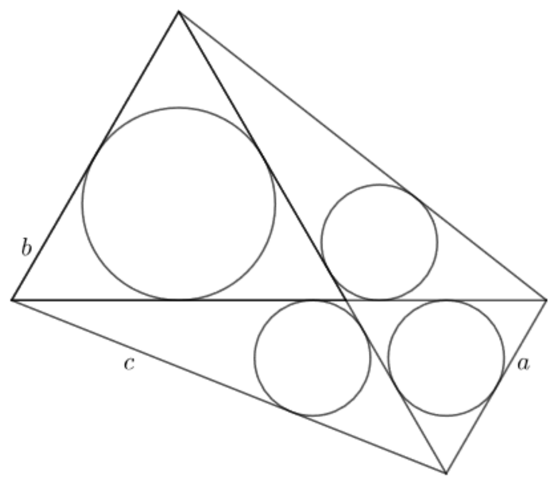

FIGURE 3.

Solution. If a, b, c are lengths of sides of a triangle, and γ is a measure of the interior angle opposite to the side with length c, then the inradius equals

$$\frac{ab\sin\gamma}{a+b+c}.$$

Thus the small circles are congruent when

$$\frac{ab\sqrt{3}}{2(a+b+c)} = \frac{a\sqrt{3}}{6}.$$

This leads to $c = 2b - a$. Substitute c into $c^2 = a^2 + b^2 + ab$, which is obtained due to law of cosines. We get $3b = 5a$ and consequently $3c = 7a$. The ratio is

$$a : b : c = 3 : 5 : 7.$$