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Three Archimedean Circles Arising from Equilateral Triangles

Emmanuel Antonio José García
Colegio Bilingue New Horizons, Ave. Sarasota, 51,
Santo Domingo, Dominican Republic.
e-mail: emmanuelgeogarcia@gmail.com

Abstract. We construct three Archimedean circles from equilateral triangles in the arbelos.

Keywords. arbelos, Archimedean circle, equilateral triangle.

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Consider an arbelos with inner semicircles AC and BC of radii a and b, and outer semicircle AB of radius $a+b$. Archimedean circles are circles in the arbelos, congruent to the Archimedean twin circles. It is known the Archimedean circles have radius equals $\frac{ab}{a+b}$ [1, 2]. In this note we construct three Archimedean circles from equilateral triangles in the arbelos.

Theorem 1. *In the arbelos construct equilateral triangles $\triangle ACD$ and $\triangle BCE$. Let AE intersect CD in F. Construct point G similarly. The circles of diameters CF, FG and GC are Archimedean (see Figure 1).*

Proof. Since the triangles $\triangle ACF$ and $\triangle ABE$ are similar,

$$\frac{CF}{AC} = \frac{BE}{AC + CB}.$$

Therefore

$$CF = \frac{AC \cdot BE}{AB} = \frac{2ab}{a+b}.$$

Similarly we have $CG = \frac{2ab}{a+b}$. Since the triangle CFG is equilateral, the circle of diameter FG is also Archimedean. □

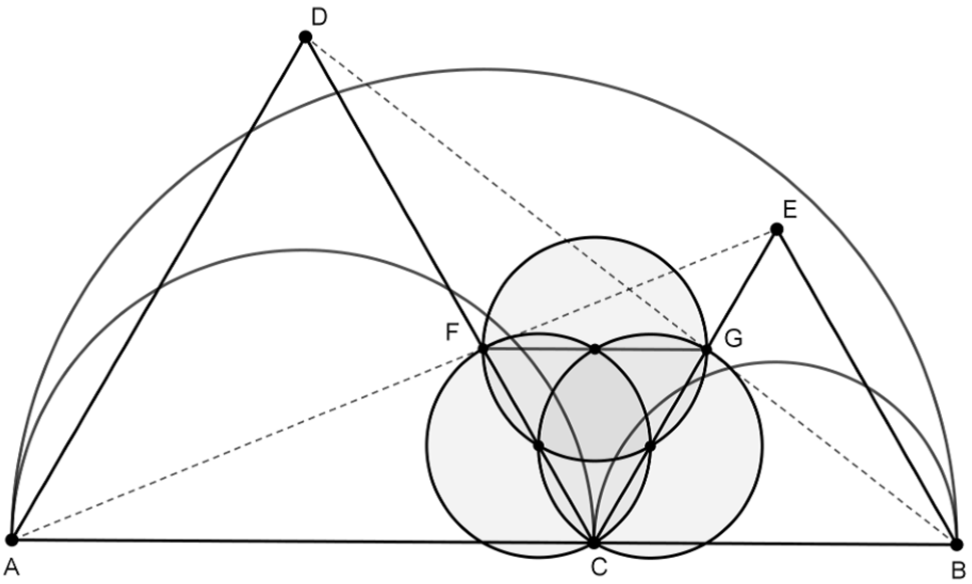

Figure 1. The circles of diameters CF, FG and GC are Archimedean.

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