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Problems 2023-1

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Send a manuscript with a simple solution of the following problems, which states something new or gives some generalization. There is no deadline of submission.

Problem 1 ([2])**.** For a rectangle $ABCD$, let α be the incircle of the triangle ABC, β is a circle lying inside of the triangle ACD and touching α and AC at the point of tangency of α and AC and touching CD, γ is a circle touching β externally and BC and CD from the inside of $ABCD$ (see Figure 1). If the circles β and γ are congruent, then show that the radius of α equals $|DA|/3$.

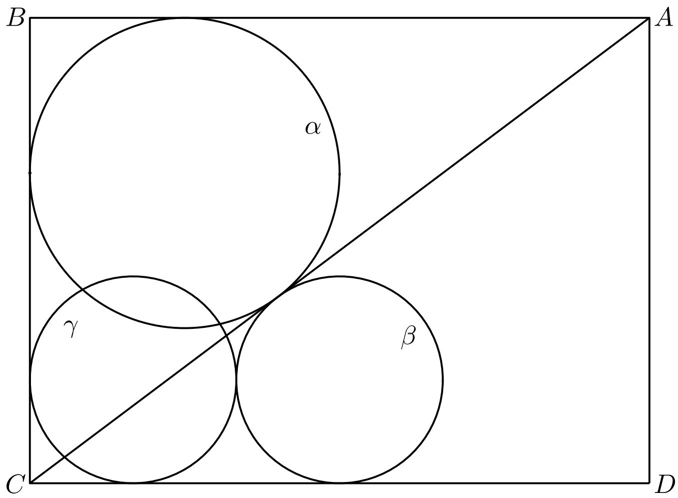

Figure 1.

Problem 2 ([1])**.** The followings are squares, where the vertices lie counterclockwise in these orders: $ABCD$, $DEFG$, $FCNH$, $GHIJ$, $INOK$, $JKLM$, $OPQL$. The point E lies on the segment CD, $a = |AB|$, $b = |DE|$, $c = |PQ|$ (see Figure 2). Show that

$$c = \sqrt{(5(a-b))^2 + (8b)^2}.$$

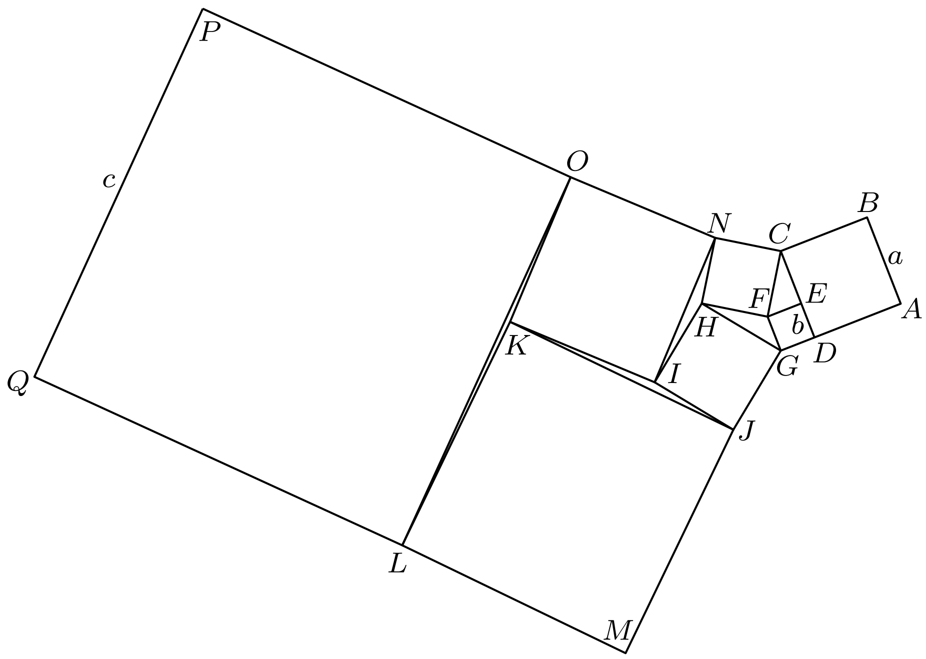

Figure 2.

Problem 3. The followings are squares, where vertices lie counterclockwise in these orders: $ABCD$, $BEFG$, $JDHI$, $CGKH$, $IKLM$, $JMON$, $OLPQ$. (see Figure 3). Say something interesting for this figure.

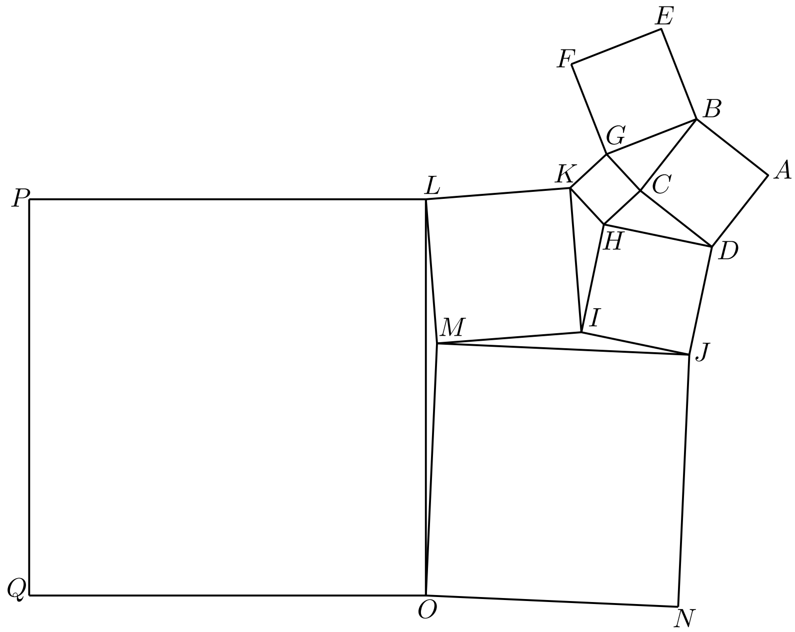

Figure 3.

Problem 4. Show $AB \perp CD$ and $|AB| = |CD|$ for Figure 4.

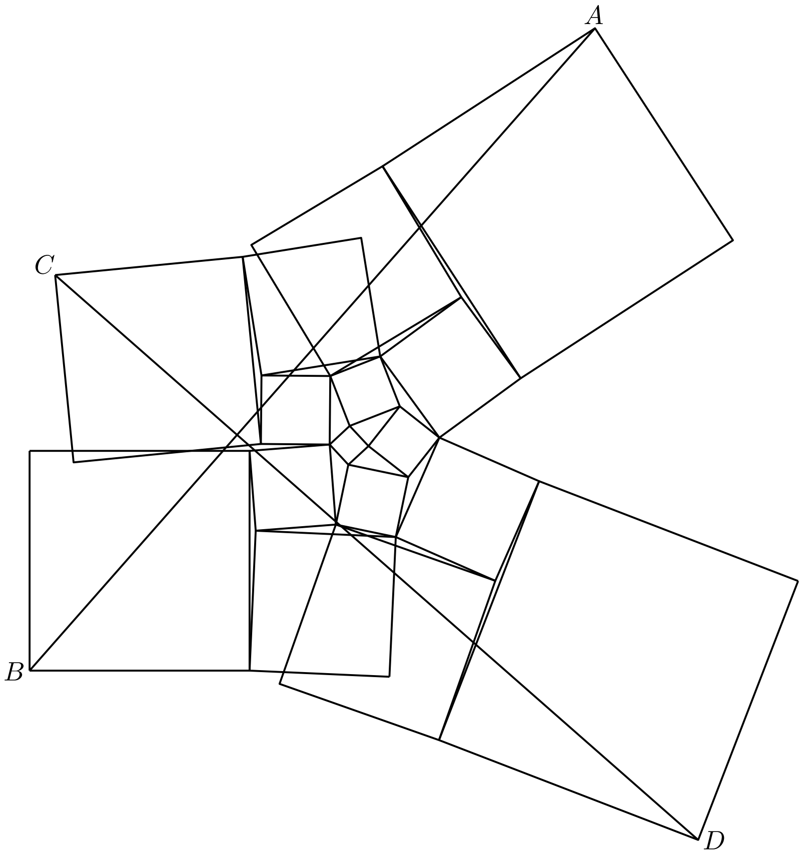

Figure 4.

Problem 5 ([2])**.** For a circle α of radius a, let AB and CD be parallel chords such that the semicircle β of diameter AB lying inside of α touches CD. If the two circles touching α internally β externally and the chord CD have radius c and are congruent to the maximal circle touching AB and the minor arc of α cut by AB, then show $a = 5c$ (see Figure 5).

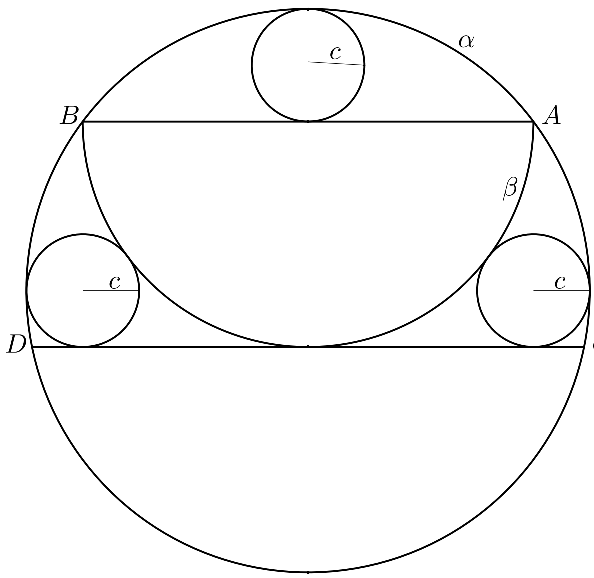

Figure 5.

Problem 6 ([1])**.** For a triangle ABC, assume that there is a circle of radius p touching CA and AB from inside of ABC and the semicircle of diameter BC externally (see Figure 6). Similarly there is a circle of radius q touching AB and

BC from inside of ABC and the semicircle of diameter CA externally. There also is a circle of radius r touching BC and CA from inside of ABC and the semicircle of diameter AB externally. Then show that the inradius of the triangle ABC equals

$$\frac{1}{2}(p+q+r+\sqrt{p^2+q^2+r^2}).$$

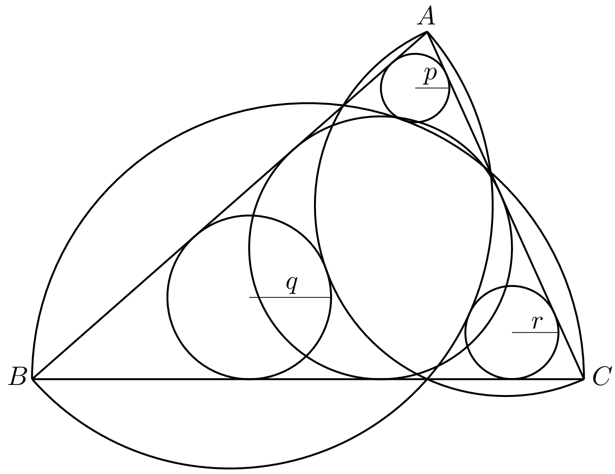

Figure 6.

References

[1] Honma (本間季隆) ed., Zoku Kanji Sampō (続勧事算法), 1849 Tohoku University Digital Collection.

[2] Kubodera (久保寺正福) ed., Kanji Sampō (勧事算法), 1821 Tohoku University Digital Collection.